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Abstract

The higher the temperature capability of the heating element, the higher is its price. However, beware of the pitfall of choosing the lowest temperature rated heating element you can purchase for your task. You could be compromising your entire profitability. Always purchase the highest rated element you can find. Why?

This article attempts to explain why. Figure 5 in the article shows the influence of temperature on your productivity. The figure shows that, the productivity, is dramatically influenced by the maximum temperature of your furnace. Note that even a slight possible increase in your furnace capability can yield enormous returns for you.

Productivity efficiencies for the processing of aluminum, iron and aluminum oxide (alumina), are given for the conditions which span heat treating to melting. Master curves for the three substances studied are shown which relate the time of processing to the material and process parameters. The results obtained from the master curves are used to calculate the relative lumped parameter productivity. A key result from the analysis indicates that any extra investment required for a higher temperature heating element when
available, offers a substantially higher monetary return on account of the dramatic productivity increase obtained with the use of the higher temperature heating element. It is found that productivity scales to the power of ten with an increase in the temperature rating of the heating element. The productivity thus appears to scale in a manner similar to diffusion processes in thermally activated systems although no such a priori assumption was made for the calculations. A typical example (table 3) for the use of the results is described where it is shown the pay back period from the purchase of a higher rated temperature heating device is much shorter than one from the purchase of a lower rated temperature heating device.

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**Introduction:**

Heating is an essential and expensive step in almost all material processing operations. The first decision which has to be made when choosing a heating device (typically a furnace or blower) is whether flame (fuel) or electric heating is to be employed. This decision is normally made by considering the local cost of available fuel, which often because of its low price/Kwhr, is able to overcome the inherent inefficiency of a fuel based direct heating system. However, electric heating is very often also the preferred heating method, because of the accuracy of temperature control possible with electric heating devices. Electric heating is also preferred over flame heating because of the environmental cost of disposing combustion effluents or from any contamination concerns from the effluents. Sometimes, electric heating is chosen to reduce the plant noise level.

When electric heating devices are considered, one of the more important and expensive constituents of the electric heating device, is the electric heating element. The prices of electric elements vary with their rated temperature capability. Typically, for elements which are able to operate in oxidizing atmospheres, the price of the element increases with the maximum rated element temperature. However, the true price of the operation is not necessarily reflected in the price of the element. A comprehensive study was reported by Evans et al (1) who determined conclusively that molybdenum disilicide
heating element furnaces were the best value (primarily because of their high surface load carrying capability). However, even within the class of molybdenum disilicide heating elements there is a wide range of available element temperature ratings (e.g. MP1700, MP1800, MP1900 etc which reflect the maximum operating temperature of the element in centigrade). Several producers worldwide manufacture electric heating elements. Typically, the life of the electric heating element in a furnace is determined by the proximity of maximum rated element operation temperature to the plant processing temperature. When used to their full rated temperatures, elements fail by creep induced thinning and melting, or by oxidation thinning and subsequent failure. Figure 1 shows a typical furnace configuration. All the heating elements described in Table 1 are schematically shown in the figure. For the calculations reported in this article a predominant five side heating a cube shaped part is considered which rests on the furnace floor.

**Table 1: Common types of electric heating element materials which can be used in air.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Metallic</td>
<td>1200°C</td>
<td>1100°C</td>
<td>Limited to 1100°C</td>
<td>Ductile</td>
</tr>
<tr>
<td>Metallic</td>
<td>1350°C</td>
<td>1300°C</td>
<td>Poor strength</td>
<td>-</td>
</tr>
<tr>
<td>Electrically heated gas</td>
<td>1300°C</td>
<td>1300°C</td>
<td>Needs airflow and temperature control.</td>
<td>Convection</td>
</tr>
<tr>
<td>Metallic</td>
<td>1650°C-2000°C</td>
<td>1600°C</td>
<td>Low Availability</td>
<td>Ductile High Surface Load</td>
</tr>
<tr>
<td>Ceramic</td>
<td>1600°C</td>
<td>1550°C</td>
<td>Elements age: brittle</td>
<td>-</td>
</tr>
<tr>
<td>Intermetallic</td>
<td>1900°C</td>
<td>1800°C</td>
<td>Brittle</td>
<td>High Surface Load</td>
</tr>
</tbody>
</table>

The question we have sought to address in this article is whether there is a correlation between the temperature of heating and the overall productivity. Productivity, for the purposes of this article, is calculated as Kg/hr of the material processed, as this reflects on the efficiency of operation, energy cost and labor hours. Conversely, the question becomes whether the added investment in heating elements which can reach a higher temperature, possibly offer a better return when compared to more economical but lower temperature capable heating elements. It is also important to know whether the magnitude of benefit if any is substantial enough to recommend the use of a higher temperature capable heating element for a given process.
Very often, for example, a decision between the use of silicon carbide heating elements (maximum temperature 1550°C) and competing molybdenum disilicide heating elements (maximum temperature 1900°C (MP1900 class)) has to be made. The plant/design engineer has hitherto fore had no real tools to make the choice except from information pertaining to the initial cost of the element or by performing the complex calculations reported in reference 1. The only instance where clarity in such a decision is available is for temperatures below 1000 °C, when the presence of convection dramatically improves the process control and efficiency (2).

After extensive numerical simulations, the results from new formulations to address the heating efficiency problem were condensed into master plots, which illustrate the relative productivity comparison for each heating situation. An illustrative example is also chosen to highlight the use of the master plots in making informed heating configuration use decisions, where the cost of the initial capital equipment becomes a major factor in the decision making process.

**Simulation**

Visualize a part being heated in a furnace with the walls radiating to a five sided cube part. The main differential equations pertinent to solve for the temperature of the part are given below. Note that the equations are highly non-linear. The lumped model utilized here is described in many texts (reference 3, for example) and the details of derivation and assumptions are listed in Appendix A. Symbols are defined below and in the list of symbols.

**MAIN DIFFERENTIAL EQUATIONS:**

\[
\frac{\partial \theta}{\partial \tau} = -N\theta^4 + C
\]

**Parameters:**

\[N = (5\sigma L/k) \times (\bar{q} \times L/k)^3\]

\[\Theta_w = (T_w \times k) \times (\bar{q} \times L)\]

\[C = N \times \Theta^4\]

\[\bar{q} = \sigma \times \varepsilon \times \left[(T_w + 273.0)^4 - (T_{amb} + 273.0)^4\right]\]

**Boundary Conditions**

At \(\tau = \tau_l\), at \(\theta = \theta_i\) to \(\theta = \theta_{\text{preset}}\); \(\tau\) has to be determined
The symbols are defined as follows: \( \theta \): dimensionless temperature, \( \tau \): dimensionless time\((\alpha/L^2)\), \( \alpha \): thermal diffusivity, \( \sigma \): Stefan-Boltzmann constant, \( \varepsilon \): Emissivity, \( L \): Length of the part, \( k \): thermal conductivity, \( t \) is the dimensional time, \( q \): heat flux, and \( T_w \) is the wall temperature. Note that the main dimensionless variables \( N \) and \( \Theta_w \) may be thought to represent the wall temperature and inverse heat flux respectively.

A calculation of the time to heat the entire part to a given temperature uniformly is determined by considering a fixed radiation temperature (corresponding to a fixed wall or heating element temperature) and then running the simulation for a choice number of different variables which include part size. The solution terminates when the preset (processing) temperature is reached. Once, the preset part temperature is achieved, electric furnaces typically regulate the power required to compensate for the loss of heat through refractories and the productivity is not further influenced in any major manner by the wall temperature.

**Results**

Initially, the simulations were carried out for obtaining plots of the time required for the part to reach the proper processing temperature with variation in \( N \), the dimensionless wall temperature and different preset temperatures (i.e., the temperature the part in the furnace has to reach). Figures 2 and 3, shows the calculated variation of the non dimensional time for processing with a variation in the non dimensional number, \( N \), for two different materials. Note that the two curves cover the range of metal treating and melting of aluminum alloys to conditions normally experienced for ceramic processing of aluminum oxide. The typical scale of the processing for metallic and ceramic parts is thus covered in the two curves. Typically, ceramic parts are smaller than metallic parts being heat-treated as is reflected in the chosen conditions for figure 3. The largest size in the order of meters is considered for simulating the conditions encountered in large aluminum melting furnaces although the calculation is not carried out for the melting heat of fusion (if melting is to be considered the heat now required is of the same order as that required to heat to the melting point and the times can be roughly doubled). Both, these figures assume an average thermal conductivity and other properties given in the figure caption.

When developing similar curves for iron, a heat of transformation \( \sim 16000\text{J/Kg} \) from ferrite to austenite phase must also be included or the specific heat modified appropriately. Figure 4, shows the calculated variation of the non dimensional time for processing iron with a variation in the non dimensional number, \( N \) for conditions which range from heat treating spring steels to typical austenitizing temperatures. In figure 4, in the boundary conditions for two cases 700\( ^{\circ}\)C and 800\( ^{\circ}\)C, the wall temperature is varied from 1150\( ^{\circ}\)C to 1500\( ^{\circ}\)C. For the 1400\( ^{\circ}\)C preset temperatures, the wall temperature is varied between 1500\( ^{\circ}\)C and 1900\( ^{\circ}\)C.
By considering $L^3$ as the volume being processed, plots are now constructed for the productivity (defined by Kg/sec of the material being processed) to reach a preset temperature. These plots are constructed from the computed master plots presented in the figures 2-4. Generally, the heating element temperatures are about 50-100°C higher than the wall temperatures in typical furnaces and so the figures offer a direct comparison of the influence of the temperature capability of heating elements on the expense of thermal processing. Note that in all cases a log linear dependence with wall temperature is noted. These plots are shown in figure 5 (a-h)

The equations relating productivity with the wall temperature for several preset temperatures are next collapsed to fit a form

$$\log(P) = a + b \cdot L + c \cdot L^2 + d \cdot T_w,$$  \hspace{1cm} (1)

The coefficients $a$, $b$, $c$ and $d$ were calculated for all three materials and are listed in Table 2.

**Table 2:** The general equation relating productivity with wall temperature for aluminum, iron and aluminum oxide is given by $\log(P) = a + b \cdot L + c \cdot L^2 + d \cdot T_w$. $P$ is in Kg/s, $T_w$, the wall temperature is in centigrade, $L$, the part size is in centimeter.

<table>
<thead>
<tr>
<th>Material</th>
<th>Temperature ($T_w$, °C)</th>
<th>a</th>
<th>b(cm$^{-1}$)</th>
<th>c(cm$^{-2}$)</th>
<th>D(°C$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum Oxide</td>
<td>800</td>
<td>-3.3144</td>
<td>0.3568</td>
<td>-1.54x10$^{-2}$</td>
<td>0.04886</td>
</tr>
<tr>
<td>Aluminum Oxide</td>
<td>1100</td>
<td>-3.4567</td>
<td>0.3306</td>
<td>-1.28x10$^{-2}$</td>
<td>0.05328</td>
</tr>
<tr>
<td>Aluminum Oxide</td>
<td>1600</td>
<td>-3.4012</td>
<td>0.3824</td>
<td>-1.73x10$^{-2}$</td>
<td>0.05453</td>
</tr>
<tr>
<td>Aluminum</td>
<td>700</td>
<td>-2.1120</td>
<td>0.04186</td>
<td>-2.20x10$^{-4}$</td>
<td>0.05177</td>
</tr>
<tr>
<td>Aluminum</td>
<td>900</td>
<td>-2.2483</td>
<td>0.04185</td>
<td>-2.20x10$^{-4}$</td>
<td>0.05369</td>
</tr>
<tr>
<td>Iron</td>
<td>700</td>
<td>-2.3255</td>
<td>0.08378</td>
<td>-8.81x10$^{-4}$</td>
<td>0.05760</td>
</tr>
<tr>
<td>Iron</td>
<td>800</td>
<td>-2.6291</td>
<td>0.08376</td>
<td>-8.80x10$^{-4}$</td>
<td>0.05875</td>
</tr>
<tr>
<td>Iron</td>
<td>1400</td>
<td>-2.2955</td>
<td>0.08388</td>
<td>-8.81x10$^{-4}$</td>
<td>0.05409</td>
</tr>
</tbody>
</table>

A summary of the anticipated gain in productivity with temperature may be summarized as follows: For aluminum (a typical metal): a 15% increase in temperature results in approximately a 65% gain in productivity, a 100% increase in length of object results in over a 250% gain in productivity (the productivity increase with size is to be simply interpreted as an increase in the furnace size and is only an incidental intuitive result). For aluminum oxide (a typical ceramic): a 15% increase in temperature results in approx. 65% gain in productivity and a 100% increase in length of object results in over 250 to 300% gain in productivity. Note that the best way to use the productivity numbers is in a relative sense i.e. to compare costs for the same processes carried out in different size furnaces or with better element performance. Appendix B shows a table which gives a detailed analysis of the productivity gain for aluminum oxide.

The productivity thus appears to scale in a manner similar to diffusion processes in thermally activated systems where an exponential scaling of parameters with temperature
is seen. Note that the governing equations to solve the thermal problem did not a priory assume any exponential or logarithmic dependence of any kind.

The price of a typical furnace is influenced by the heating elements, refractories and the device controls. A higher temperature furnace is normally more expensive and the higher price reflects the increase in the price of the heating elements and furnace refractories. Table 3 is a future time calculation of returns from a typical 12 x 12 x 14” size box shaped furnace normally employed for firing ceramic parts or for low volume heat treating of metallic parts. Note that although the price of the furnace increase with the temperature capability, surprisingly the return on the higher investment occurs in a relatively shorter period with the higher temperature capability.

The limitation of the calculations and predictions should be considered. The calculations assume that the time to reach the temperature in a furnace is rapid. Although this is true for several Metallic, Silicon Carbide, Molybdenum disilicide and Airtorch furnaces, the furnaces which have heating elements made of ceramics such as Zirconia are very slow. The furnace heat up rates may in certain instances also be deliberately set to a low value in order to prevent excessive heating speed of the surface of the part or for binder burn out purposes. The relative pay back periods calculated above will be affected by such manipulation of heat up rates although the return ratios will remain the same. In a rigorous sense the calculations are really applicable to low Biot number configurations when the temperature gradient inside the part is small. Since a lumped model was used for calculation, there was no information available on temperature gradient inside the parts. However, the Biot numbers could be calculated and the range is presented in Appendix C. Reference 4, or similar texts dealing with heat flow should be consulted to calculate the temperature gradient inside different size parts if this issue is of concern. However as pointed out in Appendix C, the gradients are minimal for low Biot numbers which is often the case.

Table 3
Relative money recovery (payback) assuming a 12 cm part. The columns reflect the rated furnace temperature and furnace price and size for the 12 cm part. The part temperature is assumed to be 1100ºC. Note that the lower temperature capable furnaces, although less expensive to initially acquire, actually become much more expensive in the life cycle of the furnace.

<table>
<thead>
<tr>
<th>Rated Temperature of the Furnace</th>
<th>1450°C</th>
<th>1600°C</th>
<th>1700°C</th>
<th>1800°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical Price (investment $)</td>
<td>($7,000)</td>
<td>($9,000)</td>
<td>($10,000)</td>
<td>($14,000)</td>
</tr>
<tr>
<td>Productivity from Figures 2 and 5</td>
<td>0.06</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Part Size, in inches</td>
<td>5”</td>
<td>5”</td>
<td>5”</td>
<td>5”</td>
</tr>
<tr>
<td>Relative time to recover the initial outlay (row 1 above).</td>
<td>2.5</td>
<td>2.0</td>
<td>1.1</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Summary

Master dimensionless plots (figures 2-4) are shown which may be used to calculate the heating times required for thermal processing in any thermal environment with a fixed temperature for any size of a part. These plots may be used for furnace selection or for solving thermal design problems. This article is the first time such plots have presented. Based on these curves a master plot for productivity as a function of part size, processing temperature (preset temperature and most importantly the available thermal environment (wall temperature) is shown in figure 5 for three important engineering materials.

The calculation to determine the amount of material processed per second in a typical furnace configuration, yield equations which indicate that the relative productivity increases by the power of ten with increasing wall temperature (and thus with the heating element temperature). As the heating element cost only marginally increases with the temperature capability, it may thus be inferred that the returns from a higher quality (more temperature capable but more expensive) heating element are considerably better than the returns from an inexpensive but lower temperature capable heating element.

References

1. J. American Ceramics Society, 81, pg 815-36, 1998
4. Transport Phenomena in Materials Processing, TMS, Warrandale, 1994

Nomenclature :

A = Surface area of the object, m².
Bi = Biot number = h L / k
c = Specific heat, J/kgK.
h_r = Radiative heat transfer co-efficient, W/m²K.
L = Length of the object
k = Thermal conductivity, W/mK.
N = Dimensionless number, (dimensionless wall temperature)
q = Reference radiative heat flux, W/m².
T = Temperature, K.
T_w = Wall temperature, K.
T_amb = Ambient temperature, K.
Appendix A

Model formulation

Object heating in a furnace is a complex process and involves conductive, convective and radiative heat transfer. Often, more than one object is heated simultaneously in a furnace and it frequently involves phase change during the thermal processing of materials. Typically, the heat is transported to the object by radiation and convection, both of which require comprehensive numerical treatment in order to assign appropriate temporal boundary condition on the object surface. Temporal evolution of temperature inside the object requires solution of complex boundary conditions, complex geometry, non-linear thermo-physical properties and phase change (references 5-7). These complex problems are best handled using comprehensive numerical model. Such models are used for both design improvements (die heating\textsuperscript{5}), and process optimization (soaking pit\textsuperscript{6} and continuous annealing\textsuperscript{7}). Please contact MHI www.mhi-inc.com for more details.

The objective of this work is to investigate the impact of raising the furnace temperature on the productivity of the furnace for wide range of materials. This is demonstrated using a simpler numerical model based on the heating of a regular-shaped object in a furnace as shown in figure 1 i.e. heated by radiation from all sides except the bottom. The following assumptions are made in the model formulation:

(1) There is no temperature gradient inside the object, i.e., the lumped model can be used.
(2) Thermo-physical properties of materials are independent of temperature.
(3) Heat of phase change is accounted through augmentation of specific heat.
(4) There is only a single object inside the furnace.
(5) Heat transport from furnace wall to the object is through radiation only.
The governing equation of a lumped model is written as,

$$\rho c V \frac{dT}{dt} = A \sigma \varepsilon (T_w^4 - T^4)$$  \hspace{1cm} (a1)

Using the following dimensionless quantities,

$$\theta = \frac{T_k}{qL}, \quad \tau = \frac{t \alpha}{L^2}, \quad q = A \sigma \varepsilon (T_w^4 - T_{amb}^4)$$  \hspace{1cm} (a2)

the governing equation takes following dimensionless form,

$$\frac{\partial \theta}{\partial \tau} = N \left( \theta_w^4 - \theta^4 \right)$$  \hspace{1cm} (a3)

where, \(N = (qL/k)^3 (\sigma \varepsilon(AL^2/Vk))\), here and \(V\) are the area and Volume respectively.

For a cube which has five surfaces exposed to radiation, \(N = (qL/k)^3 (5\sigma \varepsilon L/k)\)

**Appendix B.**

**Table B1:** Sample results on productivity gain and percentage increase in productivity as a function of size of the object and temperature (Aluminum oxide, Preset temperature 1100ºC).

<table>
<thead>
<tr>
<th>Tw</th>
<th>P at L = 2 cm ((\Delta P)_L) X 100</th>
<th>P at L = 4 cm ((\Delta P)_L) X 100</th>
<th>((\Delta P)/P) X 100</th>
<th>((\Delta P)/P) X 100</th>
<th>((\Delta P)/P) X 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>1350</td>
<td>1.27E-03</td>
<td>5.12E-03</td>
<td>3.84E-03</td>
<td>301.9</td>
<td></td>
</tr>
<tr>
<td>1400</td>
<td>1.48E-03</td>
<td>2.03E-04</td>
<td>15.9</td>
<td>16.0</td>
<td>4.46E-03</td>
</tr>
<tr>
<td>1450</td>
<td>1.70E-03</td>
<td>2.38E-04</td>
<td>14.9</td>
<td>14.9</td>
<td>5.12E-03</td>
</tr>
<tr>
<td>1500</td>
<td>1.93E-03</td>
<td>2.59E-04</td>
<td>13.4</td>
<td>8.82E-03</td>
<td>13.4</td>
</tr>
<tr>
<td>1550</td>
<td>2.19E-03</td>
<td>2.91E-04</td>
<td>12.8</td>
<td>8.82E-03</td>
<td>12.8</td>
</tr>
<tr>
<td>1600</td>
<td>2.47E-03</td>
<td>2.38E-04</td>
<td>12.8</td>
<td>9.95E-03</td>
<td>12.8</td>
</tr>
<tr>
<td>1650</td>
<td>2.78E-03</td>
<td>3.03E-04</td>
<td>12.3</td>
<td>1.12E-02</td>
<td>12.3</td>
</tr>
<tr>
<td>1700</td>
<td>3.10E-03</td>
<td>3.27E-04</td>
<td>11.8</td>
<td>1.25E-02</td>
<td>11.8</td>
</tr>
<tr>
<td>1750</td>
<td>3.46E-03</td>
<td>3.53E-04</td>
<td>11.4</td>
<td>1.39E-02</td>
<td>11.4</td>
</tr>
<tr>
<td>1800</td>
<td>3.84E-03</td>
<td>3.81E-04</td>
<td>11.0</td>
<td>1.55E-02</td>
<td>11.0</td>
</tr>
</tbody>
</table>

**Appendix C**

**Biot Number Calculation**

Since heating was radiative, Biot numbers were calculated based on following formula:
\[ Bi = \frac{h \cdot L}{k} \]

Where, \( h \): is the effective heat transfer coefficient = \( \sigma \varepsilon \left( T_w^2 + T_{amb}^2 \right) \left( T_w + T_{amb} \right) \)

- \( K \): Conductivity;
- \( L \): length,
- \( \sigma \) is the Stefan Boltzmann Constant,
- \( \varepsilon \) is the emissivity,
- The superscript \( \circ \) is the initial condition,
- \( T_w \) and \( T_{amb} \) are the wall and ambient temperature respectively in Kelvin.

For figures 2-4 the following conditions are encompassed.
- Lowest Biot number for Aluminum: is 0.025
- Highest Biot number for Aluminum: is 0.29
- Lowest Biot number for aluminum oxide: is 0.07
- Highest Biot number for aluminum oxide: is 0.87

In the iron calculation (figure 4) the heat of transformation (from alpha to gamma ~16KJ/Kg) is ignored and the specific heat is increased where applicable (see figure caption) to account for this heat. If the heat had been entirely ignored an expected error of about 10% is anticipated.

- Lowest Biot number for iron: is 0.09
- Highest Biot number for iron: is 0.43

For Biot numbers below 0.5 the lumped parameter approach may generally be thought to model conditions with a shallow temperature gradient.
A collage of different types of heating elements normally used. Please see table 1 for details.
Fig. 4: A plot of the dimensionless time to reach a preset part temperature for increasing N. This part is assumed to be steel with a thermal conductivity of 60 W/m °C, density of 7870 kg/m³. For 700 °C, a mean specific heat of 442 J/kg °C is assumed whereas for 800 °C and 1400 °C, the mean specific heat of 750 J/kg °C is assumed. The inset shows the variation considered for the length of the object (L) and the actual wall temperature (Tw) for the particular calculation. The gamut of processing from heat-treating of spring steel, to austenitizing is covered by the length scale studied.
Figure 5. Productivity curves for various materials as a function of the wall temperature. Note that the material, size (L) in cm, of the part and the preset temperature are varied in each individual curve. Note the log-linear relationship.
Expanded Figure 5(g) top and 5(c) bottom

**Iron 1400°C**

**Aluminum Oxide 1600°C**
Expanded Figure 5(a) top and Figure 5 (d)

Aluminum Oxide 800C

Wall Temperature, °C

Productivity, Kg/s

L = 2 cm
L = 4 cm
L = 8 cm
L = 12 cm
Regr. lines

Aluminum 700C

Wall Temperature, °C

Productivity, Kg/s

L = 20 cm
L = 40 cm
L = 60 cm
L = 80 cm
Plot 1 Regr